KINEMATIC ANALISYS OF MEHANISM BY COMPUTER UTILIZATION

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SUMMARY:
The following work present generalized kinematic analisys of any mechanism, namely deciding trajectory, velocity and acceleration of specific point of a certain mechanism by utilization one of the most popular software system (AutoCAD). In other words by using AutoLISP as the programming tool in AutoCAD. The work also describes simulation as the real picture of movement of a certain mechanism.

Key Words: Mechanism, movement, driving element, trajectory, hodograph, velocity, acceleration, simulation.

INTRODUCTION

It is well known that computers and computer science have influenced all spheres of human existence. This work has been made using one of the most popular applications for construction and modelling: the AutoCAD. AutoCAD contains tools for programming known as artificial intelligence programs, that is programs of the newer generation. AutoLISP is a programming language that enables us to simplify drawing and constructing, and in a very simple way automates tasks that repeat themselves many times in AutoCAD. Above-mentioned programming language is based on lists. In the USA and other English speaking countries AutoLISP is referred to as "Lost in Stupid parentheses. AutoLISP automatization can be easily applied on the mechanism theory when drawing and calculating trajectory and hodograph of velocity and acceleration. By using kinematics analysis and AutoLISP one can get the output which is impossible to get otherwise, due to the enormous number of repetitions that have to be carried out on a mechanism.

THEORETIC ANALYSIS

Mechanisms are objects that contain two or more elements that are connected in such a way that a movement of one element triggers predefined movement of other elements. Mechanisms are used for transfer of movement, or for transformation of one kind movement into another.

Kinematic analysis of a mechanism contains:

-Drawing of a position of the entire mechanism for a given position of a driving segment, or for a given segments of all driving segments, if a mechanism has freedom of movement in a number of degrees.

- Analysis of trajectory for individual points.

- Calculation of velocities and accelerations of all relevant points, angle velocities and accelerations of segments whose movement contains rotation, defining relations of velocities and acceleration with time or position of the mechanism, and drawing of velocity and acceleration graphs as a function of time or position.

Methods of kinematics analysis are numerous. Some of them are general, and can be applied to all kinds of mechanisms.

For defining velocities and acceleration, as well as angle velocities and acceleration, the following methods are used:

- Method of velocity and acceleration plan
- Method of momentary poles of rotation.
- Method of angle velocities plan
- Method of complex numbers
- Method of Assur's points, etc.

a) Velocity and acceleration plan

Basic method of velocity and acceleration plan is based on Chasles's theorem. Every body with a co-planar movement can be brought from one position to the next position by translating and rotation. The pole AB can thus be brought from A_1B_1 position into position A_2B_2, first by translating into position $A_2B_{2'}$, and then rotation around A_2, into the position A_2B_2.

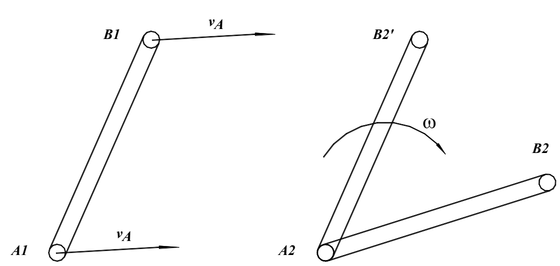


Figure 1

During the translation, all points of the pole have the same velocity, so $\overrightarrow{v_B} = \overrightarrow{v_A}$.

During the rotation of the point B around A, point B has the velocity of $\overrightarrow{v_B^A}$, whose direction is perpendicular to the radius of the pole. Since both movements are simultaneous,

$$\overrightarrow{v_B} = \overrightarrow{v_A} + \overrightarrow{v_B^A}$$

The acceleration of all points of the pole is equal during the translation, and $\overrightarrow{a_B} = \overrightarrow{a_A}$. Due to the rotation around A, the point B has the acceleration $\overrightarrow{a_B^A}$.

We can say

$$\overrightarrow{a_B} = \overrightarrow{a_A} + \overrightarrow{a_{Bn}^A} + \overrightarrow{a_{Bt}^A}$$

Where $\overrightarrow{v_B^A} = \omega \cdot \overline{AB}, \overrightarrow{a_{Bn}^A} = \omega^2 \cdot \overline{AB}, \overrightarrow{a_{Bt}^A} = \varepsilon \cdot \overline{AB}$

ω - angle velocity
ε - angle acceleration

b) Method of momentary pole rotation

When the body is moving co-planary, infinitely small movement of the body can be shown as a rotation around a point in a plane, a so called rotation pole. The Trajectory of all the points of the body are arcs whose center is in a pole of the rotation. If the body is moving in the infinetly close position in a time interval dt, the momentary pole of the rotation P is going to be positioned perpendicular to the movement velocity of the body's points. If we know the momentary poles of the rotation, we can calculate kinematics characteristics of the mechanism points. The basic method for establishing momentary poles is Kennedy's theorem: " three momentary poles of a relative movement of three bodies whose movement is co-planar are on the same direction".

For three gears on the figure 2, we have the momentary poles P_{14}, P_{24}, P_{34}, P_{12} and P_{23}, while P_{13} is on the intersection of polar directions P_{12} P_{23} and $P_{14}P_{34}$.

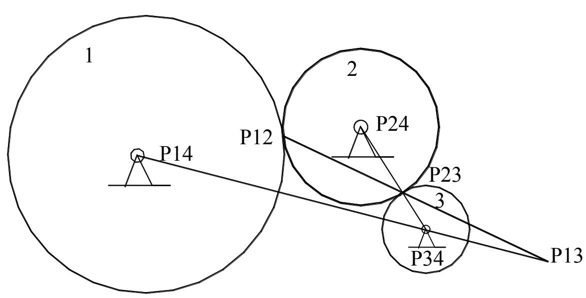


Figure 2

We can conclude based on Kennedy's theorem, that a mechanism which, consists of n members has:

P=n·(n+1)/2 poles.

The numbers of pole lines are:

$$S=P\cdot(n-2)/3=n\cdot(n-1)(n-2)/6.$$

Basically, above-mentioned tasks of kinematics analysis can be solved with one of those methods. Complex mechanisms are more often solved with graph analysis methods, because the procedure is very complex when using purely analytical method. Essentially, graph analysis of mechanism can almost always be carried out, however the procedure is very complex and time consuming, and naturally, accuracy of measurements is degraded, since they have to be calculated for the entire movement range.

If the movement of driving segment rotational with a full 360 degree angle, than the movement is usually divided twelve parts 30 degrees each, and mechanism is calculated for each individual position. Due to the complexity of calculation for kinematics measurements we are not able to calculate velocities and acceleration for each individual position of driving segment. These hodographs of velocities and acceleration of specific mechanism points are becoming insufficiently accurate. Sometimes it is very hard to plot those points while it is impossible to imagine that we can see the movement of one mechanism before we can see it in reality without the use of a computer.

ANALYSIS OF A PROBLEM

Position of a moving body can be defined with six mutually independent coordinates, which depend on time function. Free material body has six degrees of freedom, that is a movement along three coordinates, and three rotations around a coordinate. Free material body can not be the part of a mechanism, since the segments of the mechanism are moving according to pre-defined terms and they can not have six degrees of freedom.

Based on the above-mentioned definition of mechanism about strictly defined laws of movement of each individual point, we are capable of defining equalities of time and trajectories in each time segment. .

We can assume that we know the trajectory of a point in a mechanism, we can note the part of the trajectory, that is AB arc. Also, we should note right-angle coordinate system Oxyz, and we can define vector function:

$$\vec{r} = \vec{r}(t)$$

Based on this we can say that function defined in this manner satisfies continuity term, that is we can derive it at any individual point, or in any point in time.

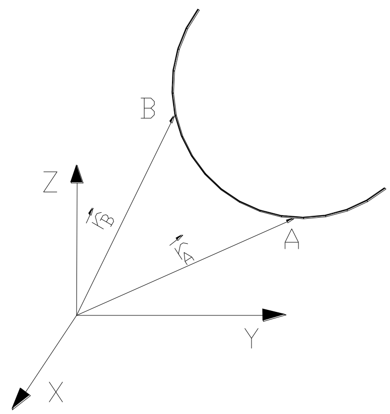


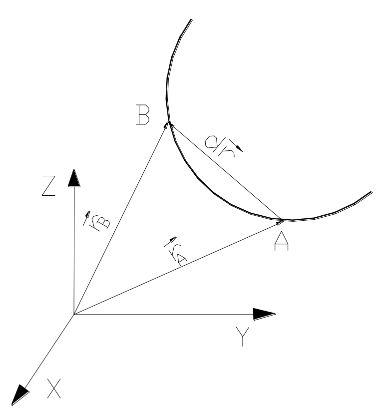
Figure 3

Lets assume that vector function **r** is continuous on an interval **α<t<β**, and that the hodograph of the function is a curve:

Limit value

$$\lim_{t \to t_0} \frac{\vec{r}(t) - \vec{r}(t_0)}{t - t_0}$$

is called the first derivative of the function $\vec{r} = \vec{r}(t)$ at a point $\mathbf{t_0}$. Derivative of the function $\vec{r} = \vec{r}(t)$ is a vector that has a direction of the tangent to the curve.



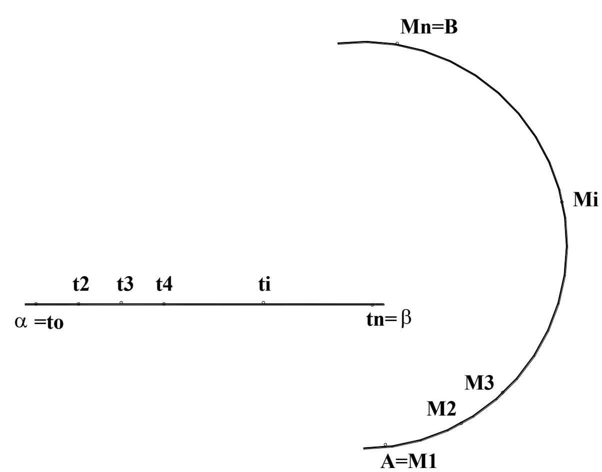


Figure 4

Figure 5

Lets look at a mechanism trajectory arc **AB** (Figure 5) that refers to time interval **$\alpha<t<\beta$**, where the point **A** corresponds to time **$t=\alpha$**, and to the point **B** corresponding time is **$t=\beta$**.

Lets look at the arc in such a way that **A** is the beginning and **B** is the end of the arc.

We divide the time interval **$[\alpha\beta]$** into **$\alpha=t_0, t_1, t_2, \ldots, t_n=\beta$.**

Lets correspond these divisions with the arc $\overset{\frown}{AB}$ divisions as follows **$A=M_0, M_1, M_2, M_3, \ldots, Mn = B$**. The point **$M_i$** corresponds to the point t_i.

Let's create the sum

$$S_n = \sum_{i=1}^{n} \overline{M}_{i-1}\overline{M}_i$$

Where $\overline{M_i M_{i+1}}$ is the length of the line whose end points are **$M_{i-1}M_i$.**

Definition: Arc AB has a length if for every positive number **e** there is a number **f (e)** so that:

$$\left|s - S_n\right| \leq e$$

As soon as:

$$\max\left(\overline{M_i M_{i+1}}\right) < f(e)$$

This method is known as rectification of the arc.

Sum **S_n** is insignificantly different from the real length, that is integral sum of the arc **s** of the function **r(t)** on a **$\alpha\beta$** segment.

Difference of these two values can be lowered at will, that is it is always smaller than **e/2** (where **e** is a small positive number), if the division of time segment is chosen in such a manner that **$\max(t_i t_{i-1}) < f(e)$.**

The same logic implies that the difference of the first derivative which is defined as

$$\underline{\lim_{t_i - t_{i-1}} \frac{\vec{r}(t_i) - \vec{r}(t_{i-1})}{t_i - t_{i-1}}}$$

Is insignificantly different from

$$\frac{\overline{M_i M_{i-1}}}{t_i - t_{i-1}}$$

Where t_i-t_{i-1} can be limited at will if the segment difference

$$\max(t_i - t_{i-1}) < f(e).$$

This implies that the first derivative at the point M_i is the ratio of length M_i M_{i-1} and of the related time segment $\Delta t = t_i - t_{i-1}$:

$$\frac{d\, r_{M_i}(t)}{dt} \approx \frac{\overline{M_i M_{i-1}}}{t_i - t_{i-1}} \approx v$$

Vrijeme	Koordinate Tačaka
t_0	M_0
t_1	M_1
…	…
t_{n-1}	M_{n-1}
t_n	M_n

Tab. 1

Based on the above-mentioned, the kinematic analysis of each mechanism only requires equations of trajectory in a table form where one column coordinate of the point, and the other is time taken for a point to reach that coordinate.

In order to get the first derivative from trajectory equality for the “every” point it is required to formulate a function in a table form using the previous one. One column would contain time interval, and the other one rectified arc lengths, that is lengths of the paths that the point covers in that particular time period.

Vremenski Intervali	Rektificirane dužine Lukova
$t_1-t_0=\Delta t_0$	$M_0M_1=D_0$
$t_2-t_1=\Delta t_1$	$M_1M_2=D_1$
…	…
$t_n-t_{n-1}=\Delta t_{n-1}$	$M_{n-1}M_n=D_{n-1}$
$t_n-t_0=\Delta t_n$	$M_nM_0=D_n$

Tab. 2

Based on this function (*Tab. 2*) vector function $\vec{v}=\vec{v}(t)$ is formed again, whose vector limits from a hodograph of velocity for a given point.

$$\vec{v}(t)=\left(\frac{D_{i-1}}{\Delta t_{i-1}}\right)\cdot\frac{\overrightarrow{D_{i-1}}}{\overline{\overline{D_{i-1}}}}, \ (i=0,1,2,\ldots,n)$$

Where, $\frac{\overrightarrow{D_{i-1}}}{\overline{\overline{D_{i-1}}}}$ - ort,

Same logic as in the previous presentation we can easily get the hodograph of the vector acceleration function.

Rectification of the vector velocity function and the first derivative we can get the new function in a table form.

Based on this function we get the vector function represented by the formula

Vemenski Intervali	Rektificirane dužine lukova
$t_1-t_0=\Delta t_o$	$V_0V_1=W_0$
$t_2-t_1=\Delta t_1$	$V_1V_2=W_1$
…	…
$t_n-t_{n-1}=\Delta t_{n-1}$	$V_{n-1}V_n=W_{n-1}$
$t_n-t_0=\Delta t_n$	$V_nV_0=W_n$

Tab. 3

$$\vec{a}(t)=\left(\frac{W_{i-1}}{\Delta t_{i-1}}\right)\cdot\frac{\overrightarrow{W_{i-1}}}{\overline{\overline{W_{i-1}}}}, \quad (i=0,1,2,\ldots,n)$$

Where, $\frac{\overrightarrow{W_{i-1}}}{\overline{\overline{W_{i-1}}}}$ - ort.

Defining of velocity and acceleration functions in a table form requires execution of an enormous number of arithmetic operations, which are not possible to obtain using standard calculating aids. For solving problems like this standard programming languages (Fortran, Pascal etc.) are of no help. Essentially, we need the programming language whose structure contains the possibility of generating huge lists, where one variable would be the value of coordinates of a curve, and which would be capable of conducting all necessary arithmetic operations on parts of the above-mentioned lists. One of specialized programming tools which has a possibility for carrying this out, and the one which is exclusively based on those lists is AutoLISP.

DESCRIPTION OF THE PROGRAM

After the program has been started, it asks for lengths of mechanism parts, the number of position of the driving element, and angle velocity of the driving element. Using given values, the program is drawing a mechanism in a given number of positions. The number of positions is up to the user, and it is entered depending on the needed accuracy of kinematic values.

For example, if we want to measure kinematic values for angle of 25,5 degrees of a driving element, then the number of positions should at least be 720, or its whole number multiple, so we can divide full 360 degrees into 0,5 degrees intervals or less.

Activating “putanje” routine, the program plots trajectories of characteristic mechanism points.

Activating “brzine” routine, the program plots velocity hodographs of characteristic mechanism points.

Activating “ubrzanja” routine, the program plots acceleration hodographs of characteristic mechanism points.

“Vrijednost” routine enables us to get values for speed and velocity of characteristic points by entering a specific angle for a driving element. (the direction as well as the value, which program draws on the hodograph).

“Kretanje” routine enables us to get simulation of motion of mechanism.

PROGRAM TESTING

Program has been modified for a two-dimensional mechanism example, which consists of (Fig. 4) rotating part OA and complex moving panel ABC and part CD, as well as the oscillating part BE, and a sliding part D, which oscillates on a horizontal straight trajectory. Rotating part OA is the driving element, which rotates in a positive mathematical direction with a constant angle velocity.

In order to carry out a program we have to load it, than enter a keyword MEH on a command prompt, as well as the length values of the mechanism L1, L2, LAC, L4, L5, H, β, f, number of positions of the mechanism, and the angle velocity of the driving part (Figure 6). The number of positions directly decides on the accuracy of kinematic values calculated by the program. Maximum number is 32767 positions. Computer’s memory and software limit it. However, 1444 positions gives us kinematics values with accuracy of 10^{-5}.

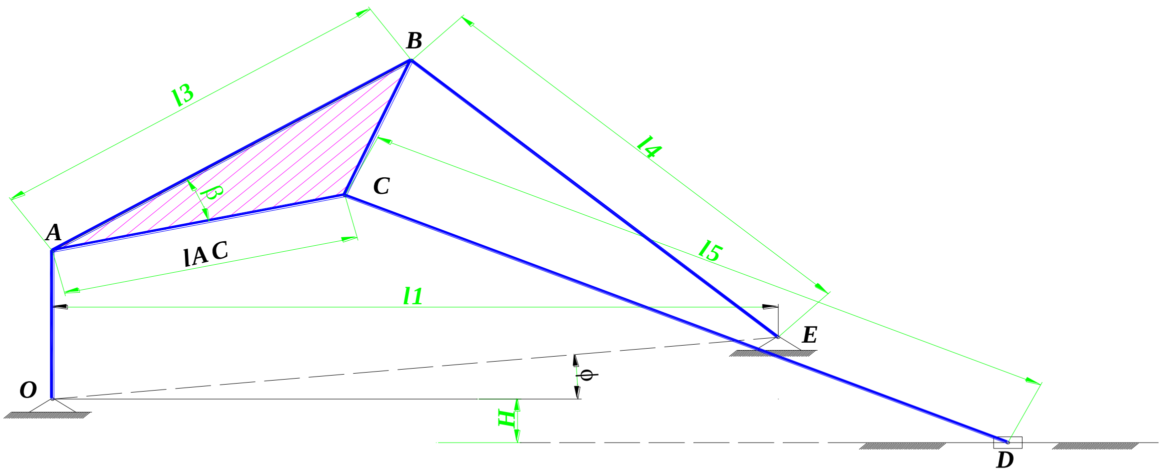


Figure 6

EXAMPLE

The Mechanism consists of (Fig. 6) rotating part OA and complex moving panel ABC and part CD, as well as the oscillating part BE, and a sliding part D, which oscillates on a horizontal straight trajectory. Rotating part OA is the driving element, which rotates in a positive mathematical direction with a constant angle velocity.

The length values of the mechanism:

L_1=0,56 (m) L_2=0,21 (m) L_3=0,52 (m)
L_4=0,35 (m) L_{AC}=0,36 (m) H=-0,07 (m)
$\varphi_1=6^0$ (deg) $\beta=-12^0$ (deg) $\omega_2=\dot{\varphi}_2=\pi$ (1/s)

$\varepsilon=\ddot{\varphi}_2=0$ $(1/s^2)$

CONCLUSION

Kinematics analysis of a mechanism with the aid of the computer, by using new programming tools is solving a number of difficulties, which are present when calculating velocity and acceleration values of a mechanism. This graphical derivation method, compared with the methods of computer aided numerical derivation shows a number of advantages. The main disadvantage of numerical derivation is the approximation of trajectory equalities by using Taylor's orders. We have to know final equations of trajectory, velocity and acceleration, and then use some of the programming languages (Pascal, Fortran etc.) for numerical derivation. Graphical method derivation requires the coordinates of trajectory points, with the density, which depends on our required accuracy. By using Newton's definition of a first derivative we can calculate velocities and acceleration. Advantages of graphical derivation and programming in AutoLISP compared to programming in Pascal or Fortran, which uses numeric derivation, is the possibility to direct error control.

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